Hand-in sheet 4 – Statistical Physics B

- *Please hand in your solution before Thursday 19 December 2024, 16:15.*
- *You can hand in your solutions in digital format as a pdf-file. Make sure to provide a file name which contains the hand-in number, your name, and your student number. You can send your solution to jeffrey.everts AT fuw.edu.pl. Also include your name and student number in the pdf file.*
- *In case of paper format, please do not forget to write your name and student number.*
- *Make sure to answer every question as completely as possible. When you do calculations, provide sufficient explanation for all steps.*
- *In total 100 points can be earned.*

Landau-de Gennes model for a nematic liquid crystal

We consider the Landau-de Gennes free energy for an uniaxial nematic liquid crystal with uniform director field,

$$f_{\mathrm{L}}(S) := \frac{F_{\mathrm{L}}(S)}{V} = f_0 + a(T - T^*)S^2 - bS^3 + cS^4,$$

with S the so-called scalar order parameter of a nematic. Here, a, b, c are positive material constants and T^* is a temperature to be interpreted in this exercise. Furthermore, f_0 is a constant independent of S.

(a) (*10 points*) What does f_0 physically represent?

(b) (*10 points*) Explain the occurrence of the cubic term in S and why it would be absent in, for example, the description of the paramagnet to ferromagnet phase transition.

(c) (*15 points*) This free energy models the transition from an isotropic liquid ($S = 0$) to a nematic state ($S \neq 0$). Show that the condition $\partial f_{\mathrm{L}}(S)/\partial S = 0$ gives rise to three branches.

(d) (*15 points*) Determine the stability of each of the branches determined in (c). In other words, determine the spinodal points of this free energy. Note that T_* is the isotropic spinodal temperature.

(e) (*20 points*) Show that the *global* minimum condition for $f_{\mathrm{L}}(S)$ results in

$$S(T) = \begin{cases} \dfrac{3b}{8c}\left[1 + \sqrt{1 - \dfrac{8(T - T_*)}{9(T_{IN} - T_*)}}\right], & (T < T_{\mathrm{IN}}), \\ 0, \quad (T > T_{\mathrm{IN}}). \end{cases}$$

(f) (*20 points*) Denote the nematic spinodal temperature by T_+ and the corresponding order-parameter value by S_+. Plot all stable, unstable and metastable solutions S/S_+ as function of $(T - T_*)/(T_+ - T_*)$. Indicate also in your plot the spinodal and binodal points.

(g) (*10 points*) Consider an isotropic liquid ($T > T_{\mathrm{IN}}$) that we quench into the nematic phase ($T < T_{\mathrm{IN}}$). What is the difference when we quench into a system with temperature $T_* < T < T_{\mathrm{IN}}$ compared to $T < T_*$?